Equitable total coloring of corona of cubic graphs

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Total coloring

A *k-total-coloring* of G is an assignmant of k colors to the vertices and edges of G in such a way that incident or adjacent elements cannot be colored with the same color. The least number of colors admitted such coloring is named the *total chromatic number* and denoted by $\chi''(G)$.

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Equitable total coloring

The proper k-total-coloring of G is said to be *equitable* if the number of elements (vertices and edges) in any two color classes differ by at most one. The minimum number of colors admitted equitable total coloring of G is named equitable total chromatic number and denoted by $\chi''_{=}(G)$.

Total Coloring Conjecture; Behzad 1965, Vizing 1968

For every graph G with maximum degree Δ, we have

$$\Delta + 1 \leq \chi''(G) \leq \Delta + 2.$$

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Equitable Total Coloring Conjecture; Fu 2002

$$\Delta(G) + 1 \leq \chi''_{=}(G) \leq \Delta(G) + 2$$

Wang 2002

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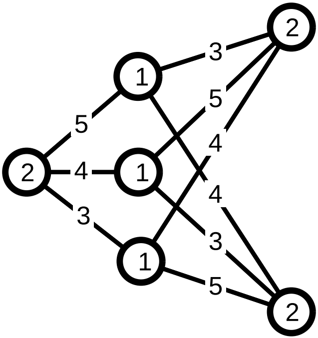
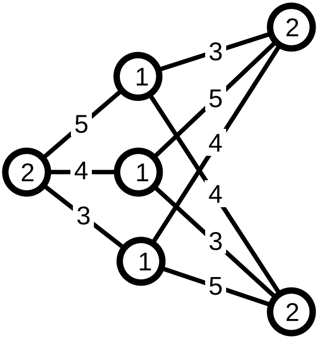
Sasaki et al. 2014

The problem of deciding if $\chi''_{=}(Q) = 4$ is NP-complete.



$$\chi''(K_{3,3}) = \chi''_{=}(K_{3,3}) = 5$$



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There are cubic graphs Q such that

$$\chi''(Q) = \Delta + 1 < \chi''_{=}(Q) = \Delta + 2.$$

Corona $G \circ H$; Frucht and Harary (1970)

The *corona* of two graphs G and H is the graph $G \circ H$ formed from one copy of G, named the *center graph*, and $|V(G)|$ copies of H, named *outer graph*, where the ith vertex of G is adjacent to every vertex in the ith copy of H.

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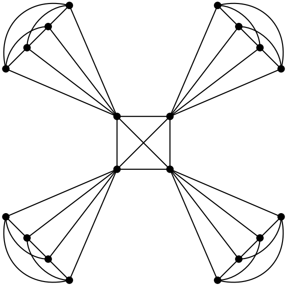

$K_4 \circ K_4$

Main result

Let G and H be cubic graphs on n_G and n_H vertices, respectively. Then

$$\chi''_{=}(G \circ H) = \Delta(G \circ H) + 1 = n_H + 4.$$

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Graph $G \circ N_{n_H}$ without pendant vertices has equitable total $(n_H + 4)$-coloring.

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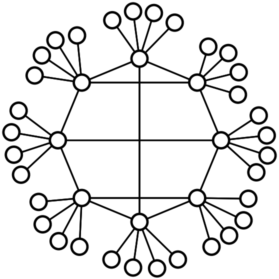

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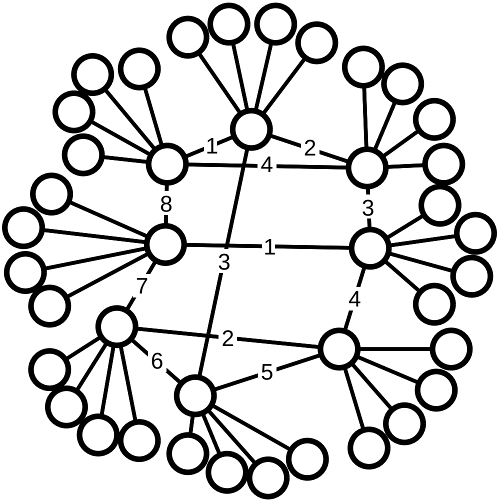
   It is possible due to Theorem [Furmańczyk, 2005]: *Every graph G has an equitable edge k-coloring for each $k \geq \chi'_{=}(G)$.*



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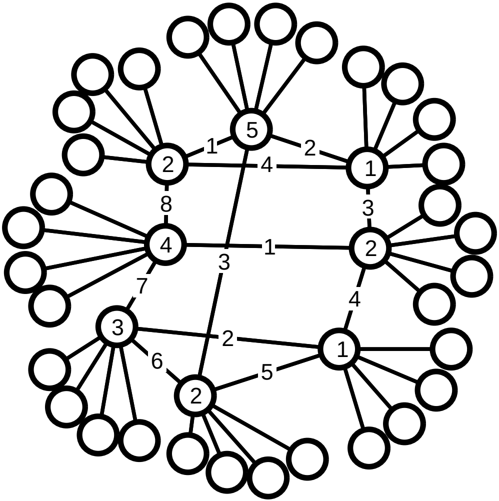
2. Color vertices of G in any proper way (with max. $n_H + 4$ colors).



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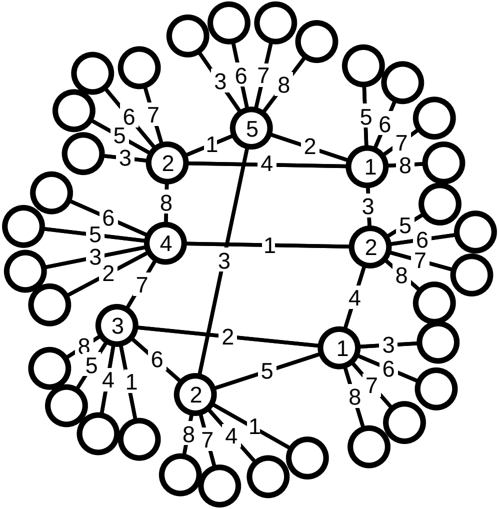
2. Color vertices of G in any proper way (with max. $n_H + 4$ colors).
3. Color pendant edges with $n_H + 4$ colors.


3 6 7 8
6 7 5 6
5 5 7
1 2
3 2 4 1 8
8 3
6 5
4 5 6
3 1 2 7
3 8
2 7 4
7
3
8 2 1 3
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Step 2

Extend the partial total coloring into copies of H in $G \circ H$: H_i, $1 \leq i \leq n_G$

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Corollary

Let G and H be r-regular graph on n_G and n_H vertices, respectively. Then

$$\chi''(G \circ H) = \Delta(G \circ H) + 1 = n_H + r + 1.$$

THANK YOU FOR YOUR ATTENTION