Model checking and interval temporal logics

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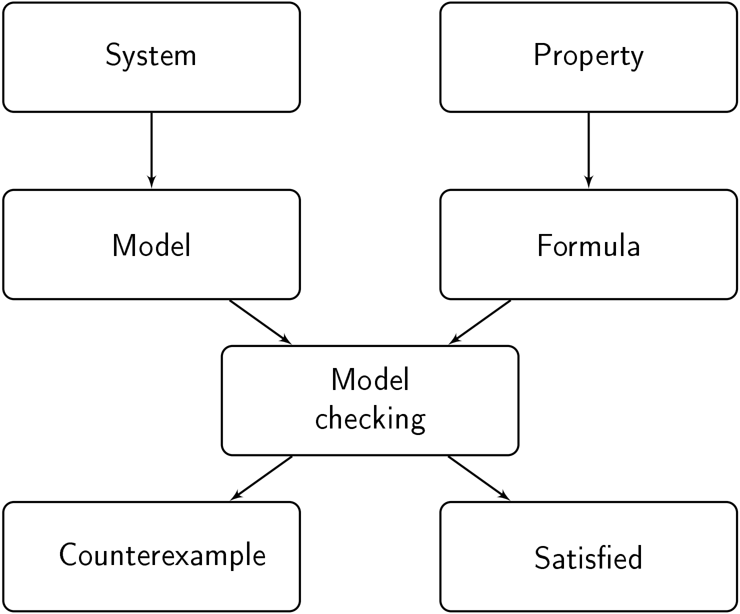
Model-Checking Multi-Agent Systems

- Leading verification technique to verify complex systems.
- Formulas are always evaluated at a state.



Model-Checking Multi-Agent Systems

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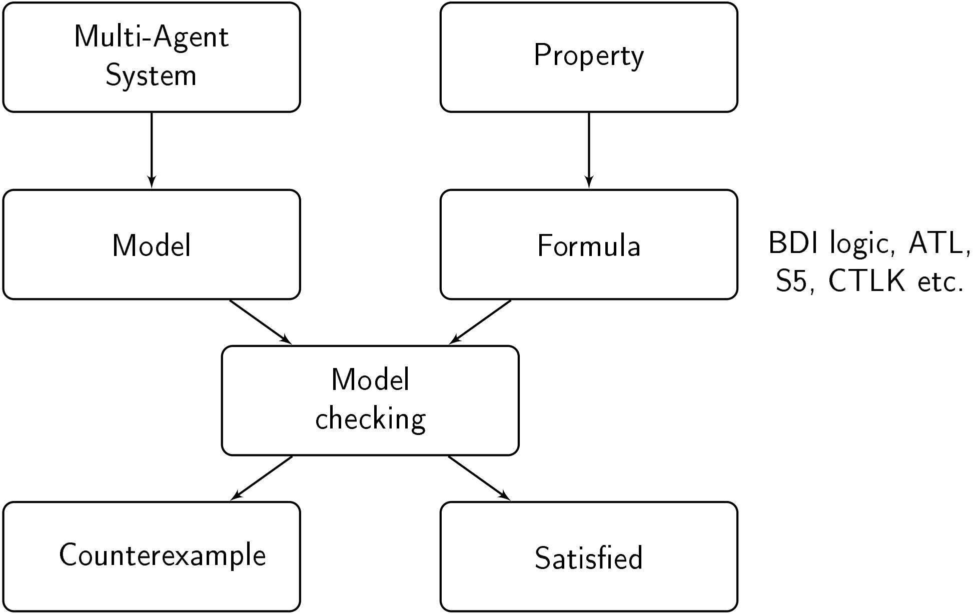
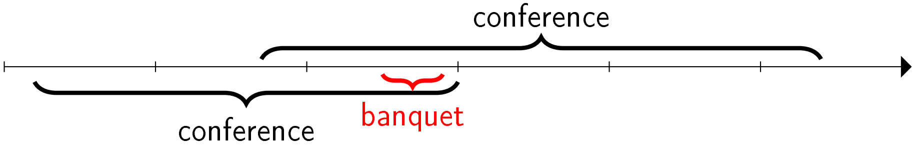
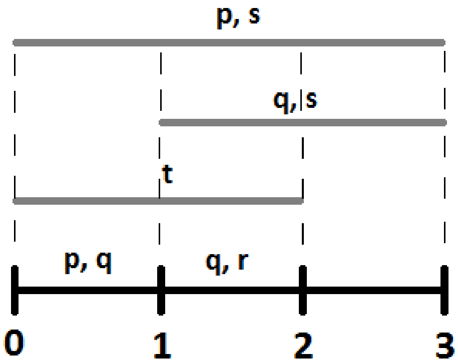


- Consider a property "there is always a banquet during a conference"
- HS: $[Globally](conference \Rightarrow \langle During \rangle banquet)$
- LTL: $G(conference_{starts} \rightarrow X(\neg conference_{ends} U(banquet_{starts} \wedge (\neg conference_{ends} U(banquet_{ends} \wedge \neg conference_{ends} \wedge F conference_{ends})))))$



Formally - the models (agent-free)

- Any order $\mathcal{D} = \langle \mathbb{D}, \leq \rangle$
- Set of propositional variables $\mathcal{V}ar$
- Classic temporal logics — labelling $\gamma : \mathbb{D} \to \mathcal{P}(\mathcal{V}ar)$
- Interval temporal logics — labelling $\gamma : \mathrm{I}(\mathbb{D}) \to \mathcal{P}(\mathcal{V}ar)$, where $\mathrm{I}(\mathbb{D}) = \{[a, b] | a, b \in \mathbb{D} \wedge a \leq b\}$



Formally - the models (with agents)

Interpreted Systems with Regular Labelling (ISRL)

ISRL is a tuple $M = (S, s_0, t, L, \{\sim_i\}_{i \in A})$, where

- The set S is the set of reachable global states,
- $s_0 \in S$ is an initial global state,
- $t \subseteq S^2$ is the transition relation,
- $L : Var \to RE_S$ is a labelling function; $L(p)$ is a **regular expression** over states defining the set of intervals satisfying p,
- $\sim_i \subseteq S^2$ is an equivalence relation.

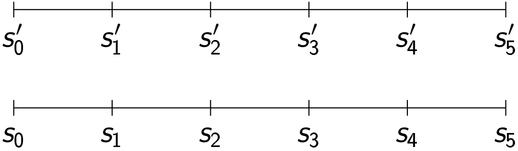
If $s \sim_i s'$, then we say that agent i cannot distinguish between states s and s'.

Intervals – paths in models*

*strictly speaking, we unravel the models into trees

Epistemic Interval Temporal Logic

- Epistemic Interval Temporal logic, EIT, propositional logic with epistemic modalities K_i ("agent i knows that") and C_G ("it is a common knowledge in G that")
- $\varphi ::= p \mid \neg\varphi \mid \varphi \land \varphi \mid K_i\varphi \mid C_G\varphi$
- $M, I \models K_i\varphi$ if in all the intervals I' that are indistinguishable for i from I φ holds.



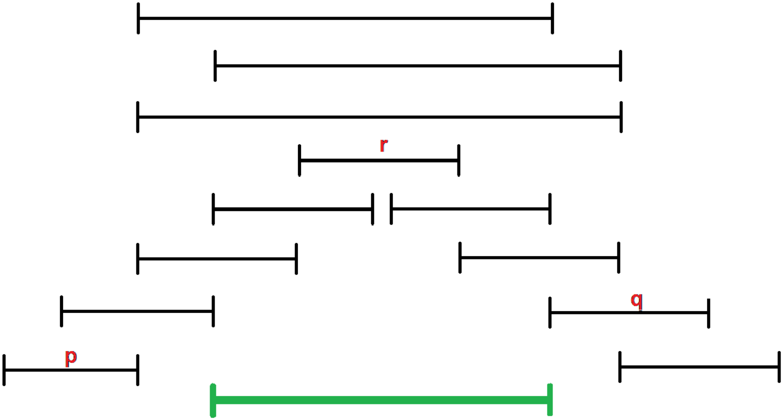
- $s_0 \ldots s_l \sim_i s'_0 \ldots s'_l$ iff for all $k \leq l$, $s_k \sim_i s'_k$.

Results for EIT [unpublished; a weaker version appeared in Lomuscio and Michaliszyn, IJCAI 2013]

Model-checking problem

- Given a ISRL $\mathcal{M}$, an interval I of $\mathcal{M}$, and an EIT formula φ.
- Question: Is φ satisfied in I?
- The problem is PSPACE-complete.

Halpern–Shoham logic

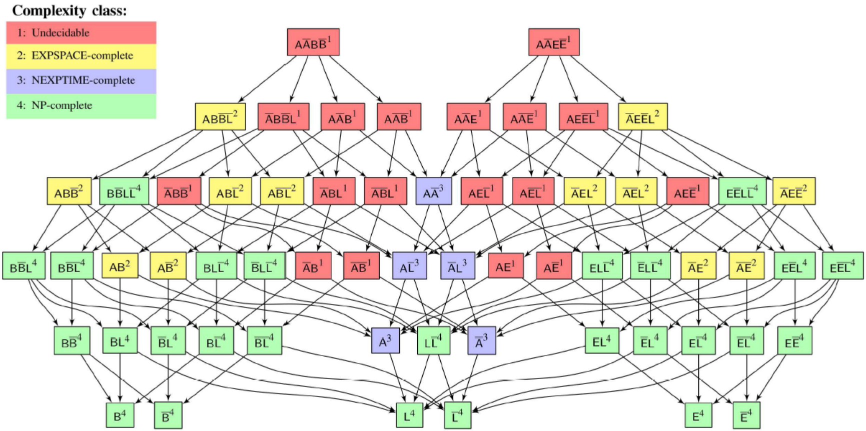


$$\langle before \rangle p \wedge \langle during \rangle (r \wedge \langle later \rangle q)$$

The Halpern–Shoham Logic

- Halpern–Shoham logic contains 12 operators $A, \bar{A}, B, \bar{B}, D, \bar{D}, E, \bar{E}, L, \bar{L}, O, \bar{O}$
- Halpern–Shoham (LICS'86) — undecidability (of the satisfiability problem for the formulas of the HS logic) using "after", "begins" and "ends".
- A lot of undecidabile fragments:
  - BE, $B\bar{E}$, $\bar{B}E$, $\bar{B}\bar{E}$, O, $\bar{O}$, AD, $A\bar{D}$, $\bar{A}D$, $\bar{A}\bar{D}$.
- Decidability of the $B\bar{B}D\bar{D}L\bar{L}$ over dense orders.

The complete picture



Main contributions of the paper

To prove that the diagram is correct (assuming strong discreteness)

Epistemic Halpen–Shoham logic

EHS: EIT with all the Halpern–Shoham modalites.

$$sleep \Rightarrow \langle A\rangle(K_{Jakub} dream \wedge \neg K_{Jakub}\langle A\rangle K_{Jakub} dream)$$

Theorem (Marcinkowski and Michaliszyn, LICS 2011)

The satisfiability problem for EHS is undecidable, even if D is the only allowed modality.

BDE fragment of *EHS* is a syntactic fragment that consists of all the knowledge modalities and $\langle B\rangle$ (begins), $\langle D\rangle$ (during), and $\langle E\rangle$ (ends).

Theorem (unpublished; a weaker version appeared in Lomuscio and Michaliszyn, IJCAI 2013)

The model checking problem for the BDE fragment of EHS is PSPACE-complete.
Better complexities by adding more constraints (e.g., on the knowledge depth of formulas).

More interesting fragment

Theorem (unpublished; a weaker version appeared in Lomuscio and Michaliszyn, ECAI 2014)

The model checking problem for the $A\bar{B}L$ fragment of EHS is decidable.

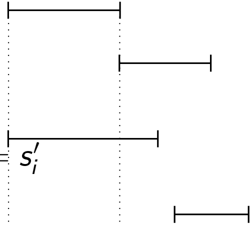
$A\bar{B}L$ fragment

$$\varphi ::= p \mid \mathrm{pi} \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid C_G\varphi \mid \langle A\rangle\varphi \mid \langle \bar{B}\rangle\varphi \mid \langle L\rangle\varphi$$

where $p \in Var$, $i \in A$ is an agent, and $G \subseteq A$ is a set of agents.

- $M, I \models p$ iff $I \in Lang(L(p))$.
- $M, I \models \mathrm{pi}$ iff I is a point interval.
- ...
- $M, I \models \langle X\rangle\varphi$ iff there exists an interval I' such that IR_XI' and $M, I' \models \varphi$, where R_X is an Allen's relation as below.

$s_1 \dots s_k R_A s'_1 \dots s'_l$
  iff $s_k = s'_1$
$s_1 \dots s_k R_{\bar{B}} s'_1 \dots s'_l$
  iff $l > k$ and $\forall i \leq k, s_i = s'_i$
$s_1 \dots s_k R_L s'_1 \dots s'_l$
  iff $t^+(s_k, s'_1)$



$A\bar{B}L$ fragment semantics

Standard semantics:

- $M, I \models \langle X \rangle \varphi$ iff there exists an interval I' such that $I R_X I'$ and $M, I' \models \varphi$, where R_X is an Allen's relation.

Bounded semantics:

- $M, I \models \langle X \rangle \varphi$ iff there exists an interval I' such that $I R_X I'$, $|I'| < |I| + f^M(\varphi)$ and $M, I' \models \varphi$, where R_X is an Allen's relation as below.

$f^M(\varphi)$ is a recursive, non-elementary function.

Theorem

Bounded semantics and standard semantics are equivalent.

Conclusion

The model checking problem for the $A\bar{B}L$ fragment of *EHS* is decidable.

Summary

- The model checking problem for the $A\bar{B}L$ fragment of EHS is decidable.
- The technique can be extended to the $A\bar{A}\bar{B}\bar{D}\bar{E}L\bar{L}$ fragment.
- Further work: to find a better upper bound.

Thank you for your attention!