Efficient Separability of Regular Languages by Subsequences and Suffixes

Wojciech Czerwiński

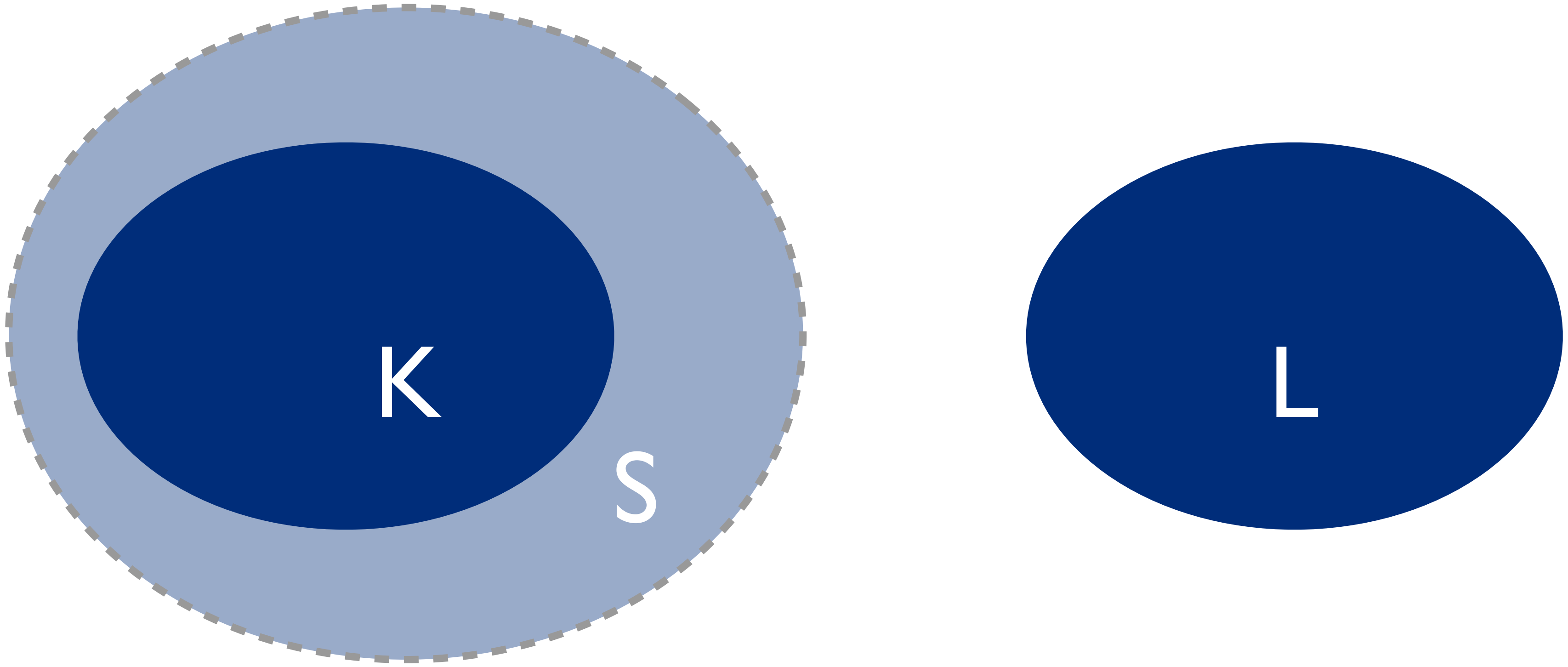
Tomáš Masopust

Wim Martens

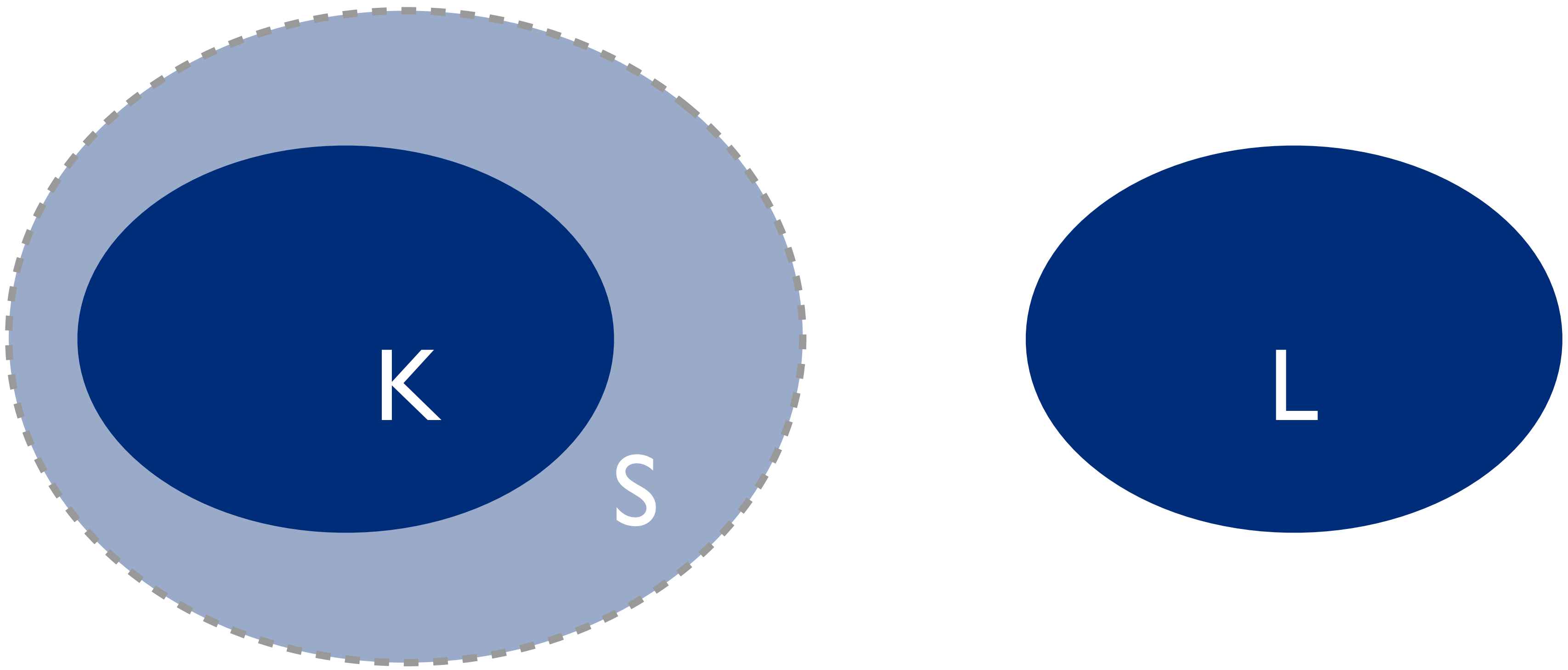
Separability

Separability

Separability

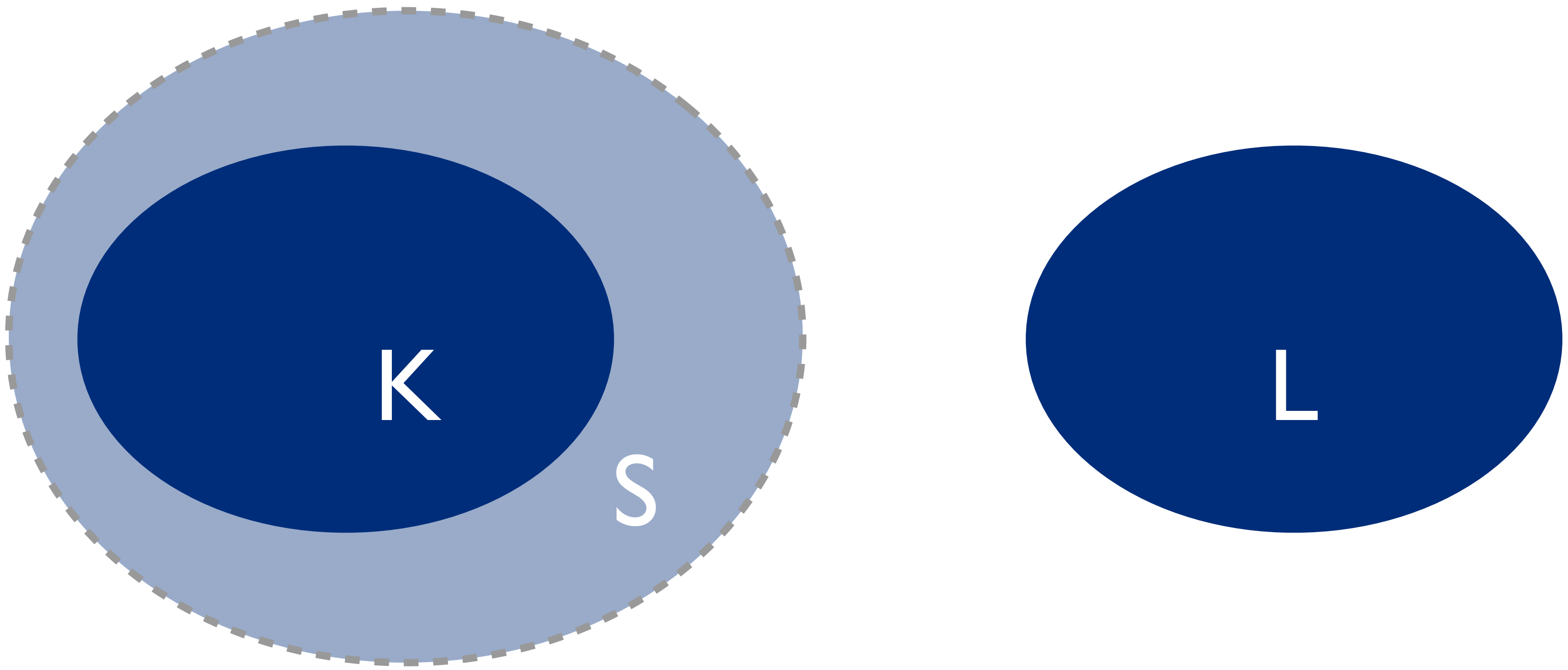


Separability



S *separates* K and L

Separability



S *separates* K and L

K and L are *separable by* family F
if some S from F separates them

Problem

Problem

Given: nondeterministic automata for languages K and L

Problem

Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

Problem

Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

piece language

Problem

Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

Problem

Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

piecewise testable language

Problem

Given: nondeterministic automata for languages K and L

Question: are K and L separable by Piecewise Testable Languages (PTL)?

piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

piecewise testable language bool. comb. of pieces

What is known?

What is known?

- Simon 1975, piecewise testable = j-trivial

What is known?

- Simon 1975, piecewise testable = j-trivial
- Stern 1985, deciding if a language is piecewise testable is in PTIME

What is known?

- Simon 1975, piecewise testable = j-trivial
- Stern 1985, deciding if a language is piecewise testable is in PTIME
- Almeida, Zeitoun 1997, exponential algorithm for separability by PTL

First main result

First main result

Theorem:
Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

First main result

Theorem:
Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

obtained independently
by Place, van Rooijen, Zeitoun
MFCS `13

First main result

Theorem:
Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

Second main result

Second main result

For any two word languages K and L the following conditions are equivalent:

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

Infinite zigzag

Infinite zigzag

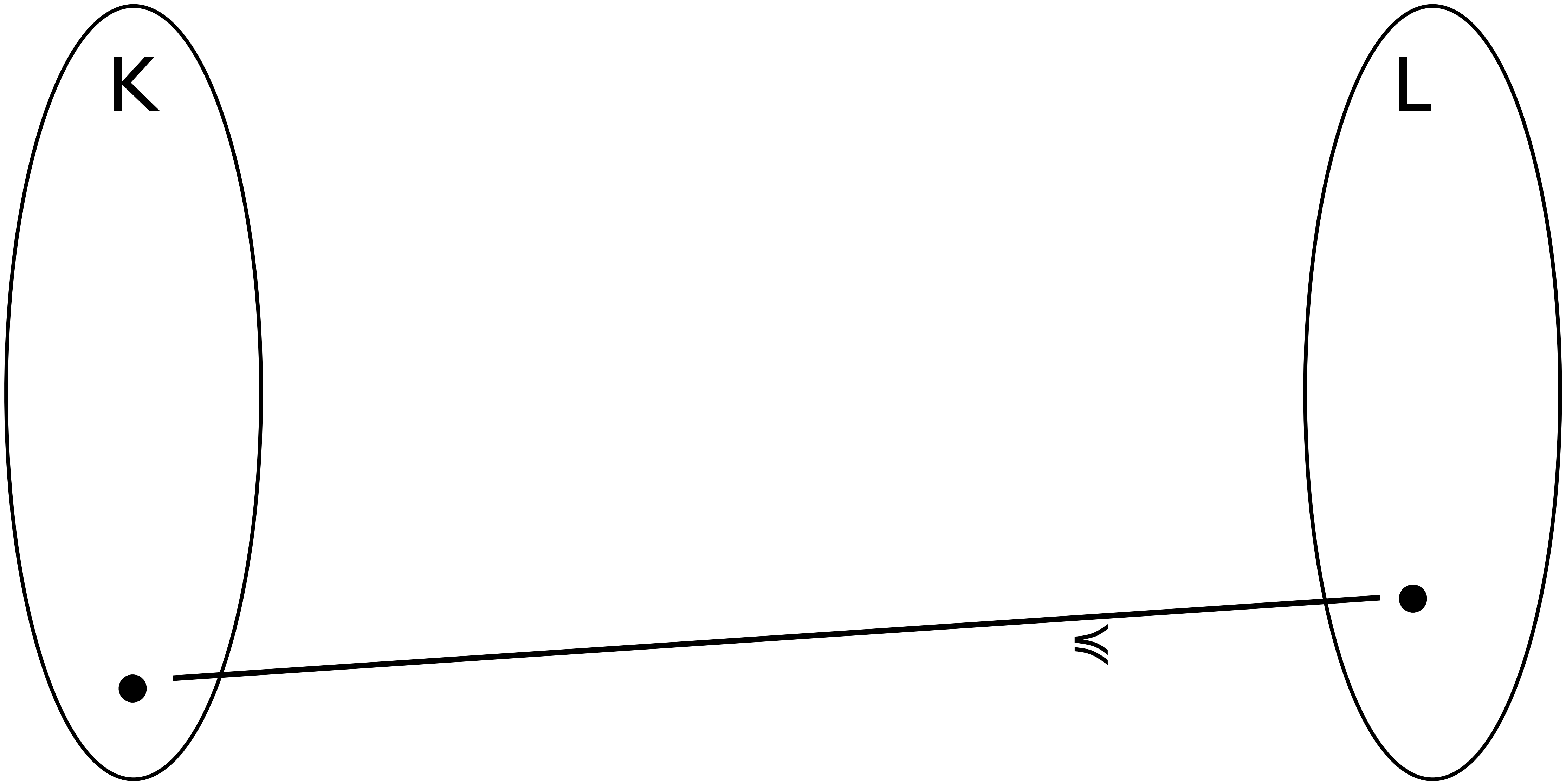
abcd ≼ dbabacbcdb

Infinite zigzag

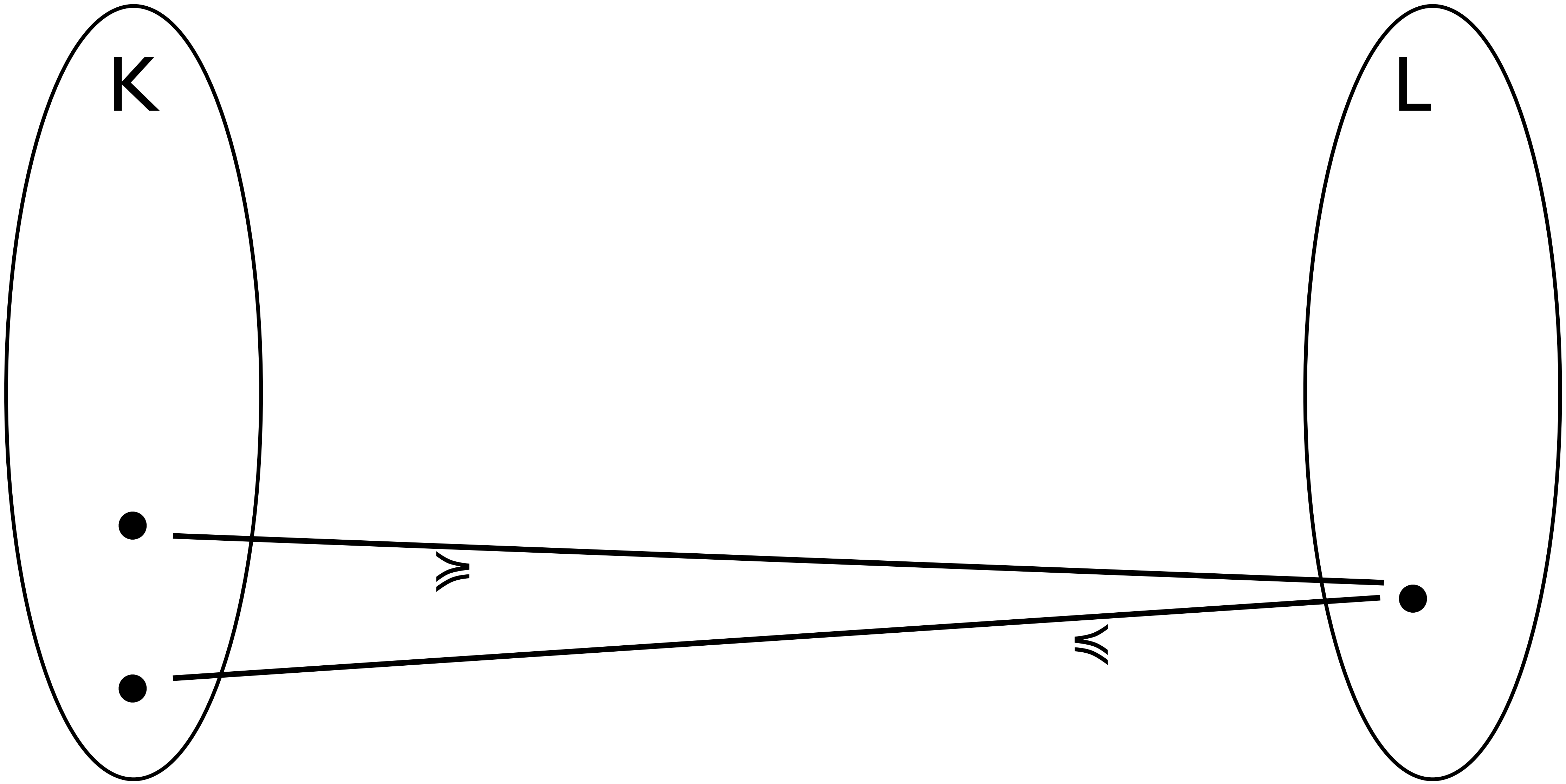
Infinite zigzag

Infinite zigzag

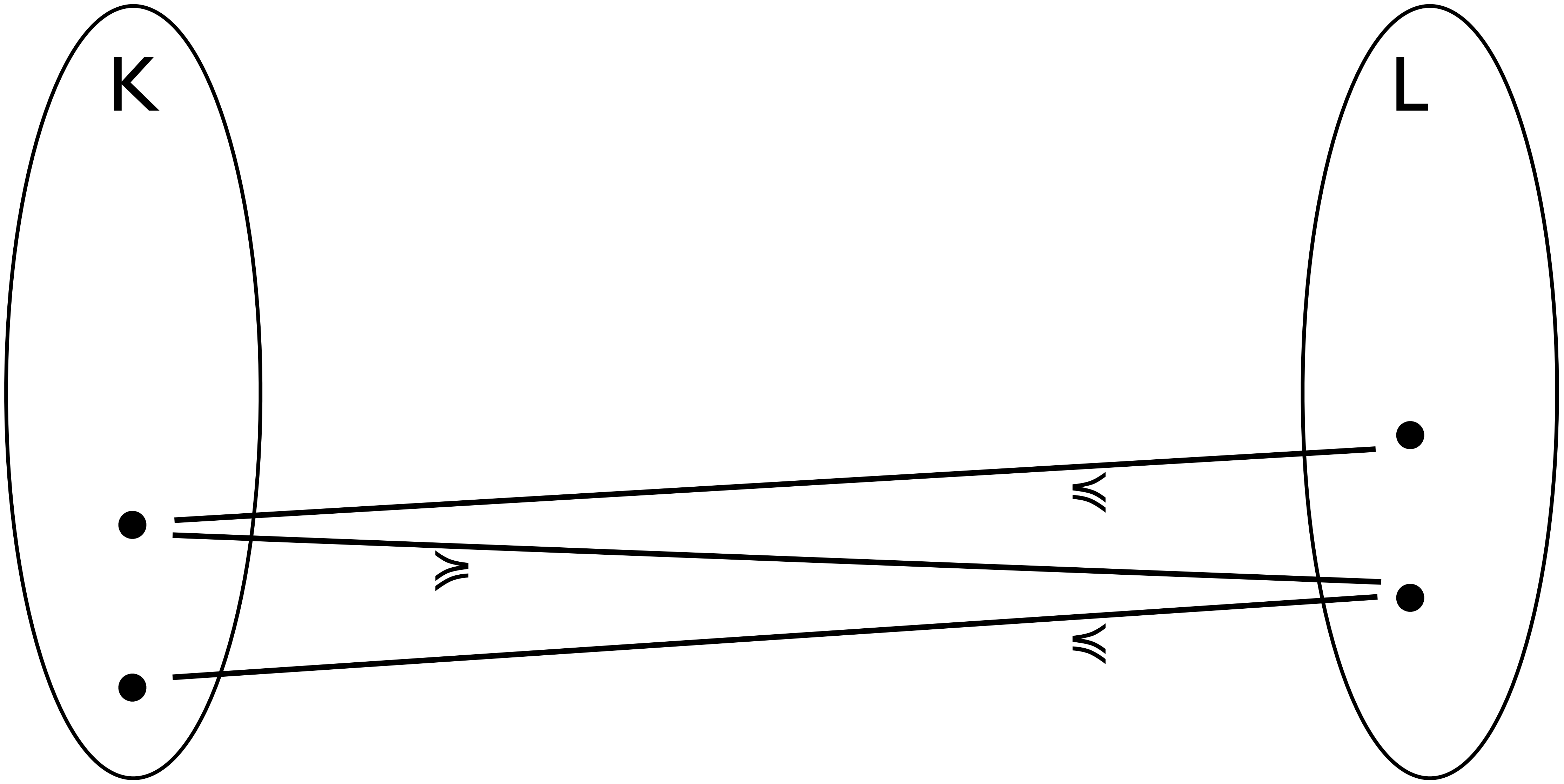
Infinite zigzag



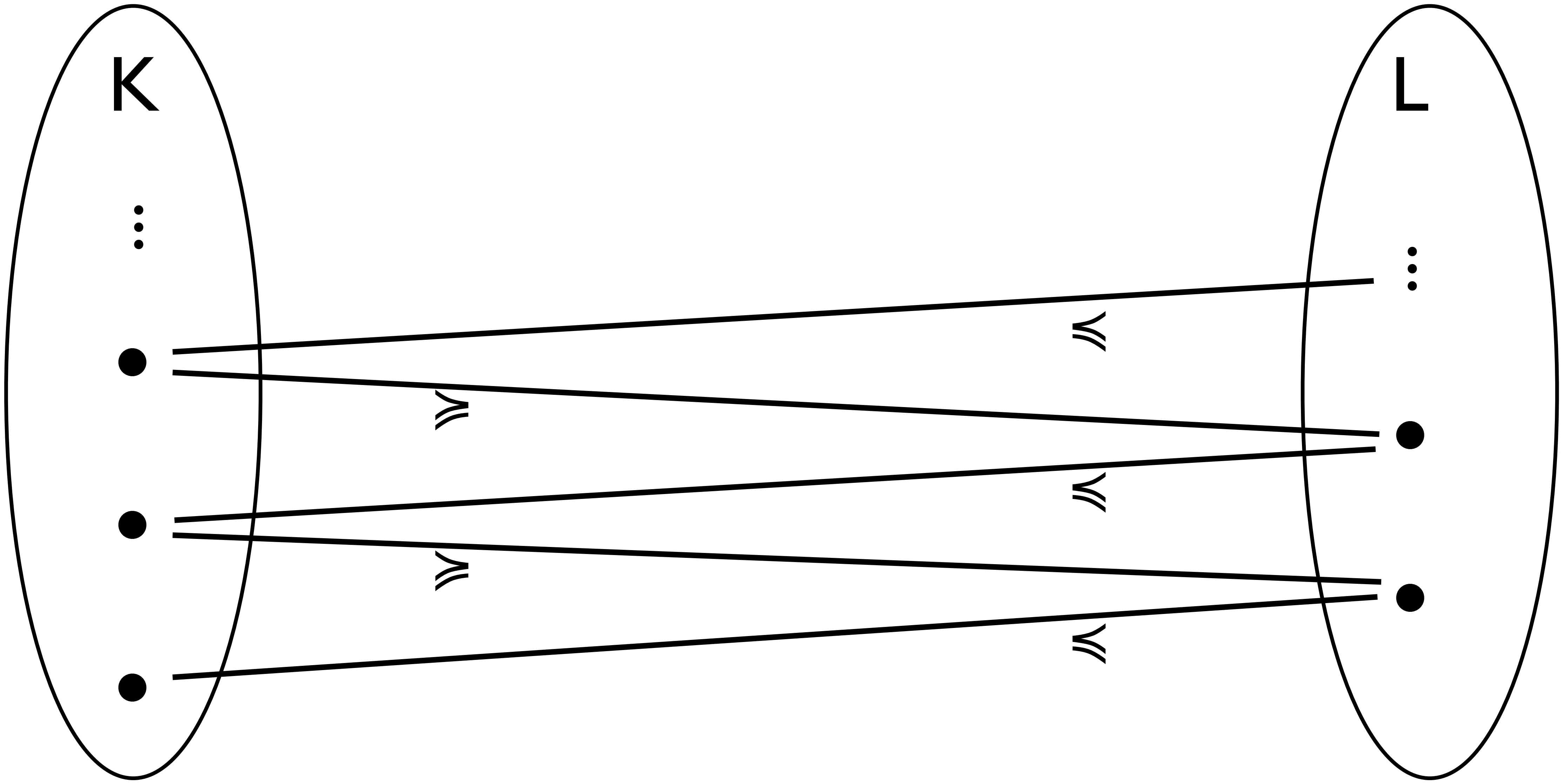
Infinite zigzag



Infinite zigzag



Infinite zigzag



Second main result

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

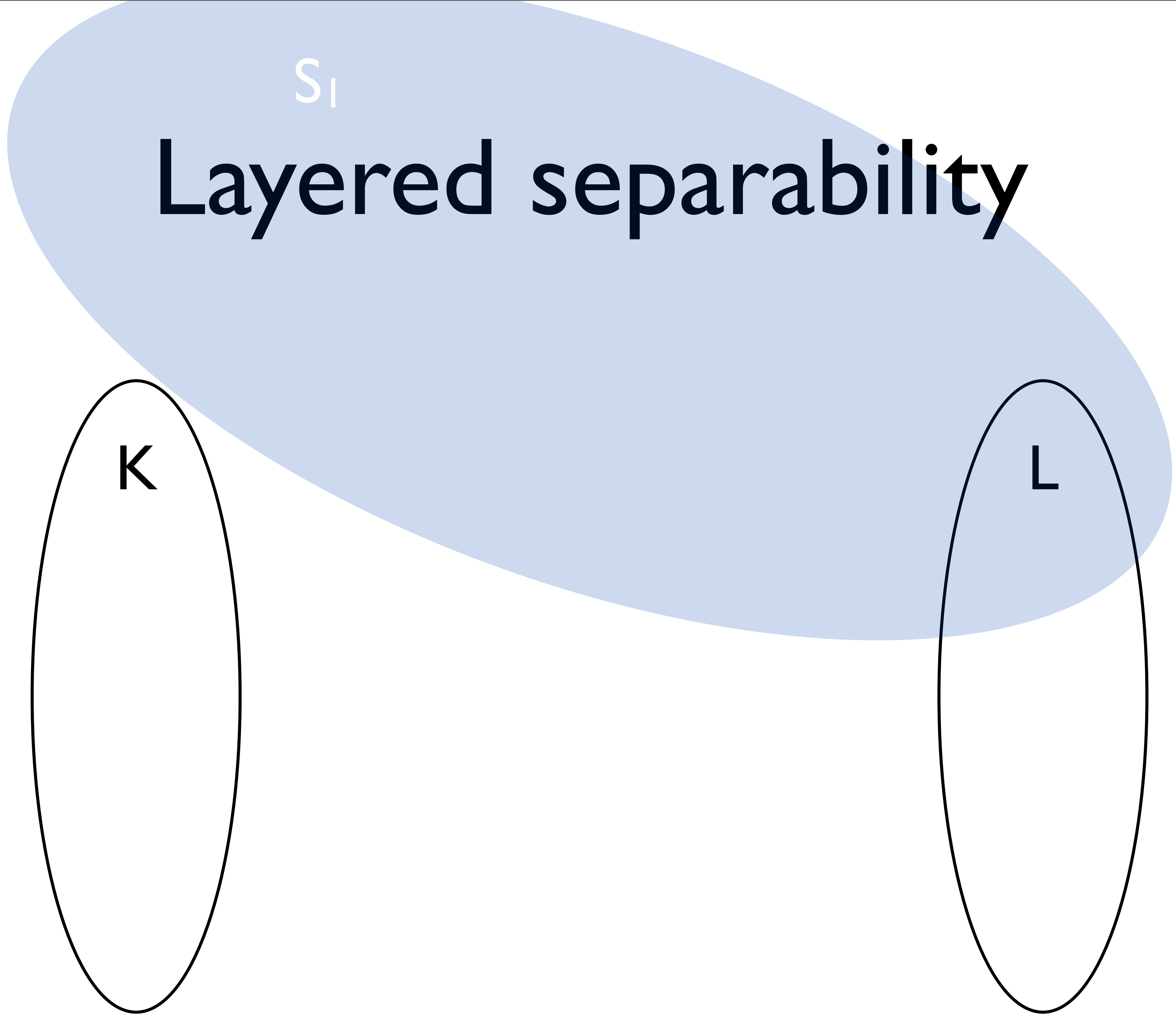
2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

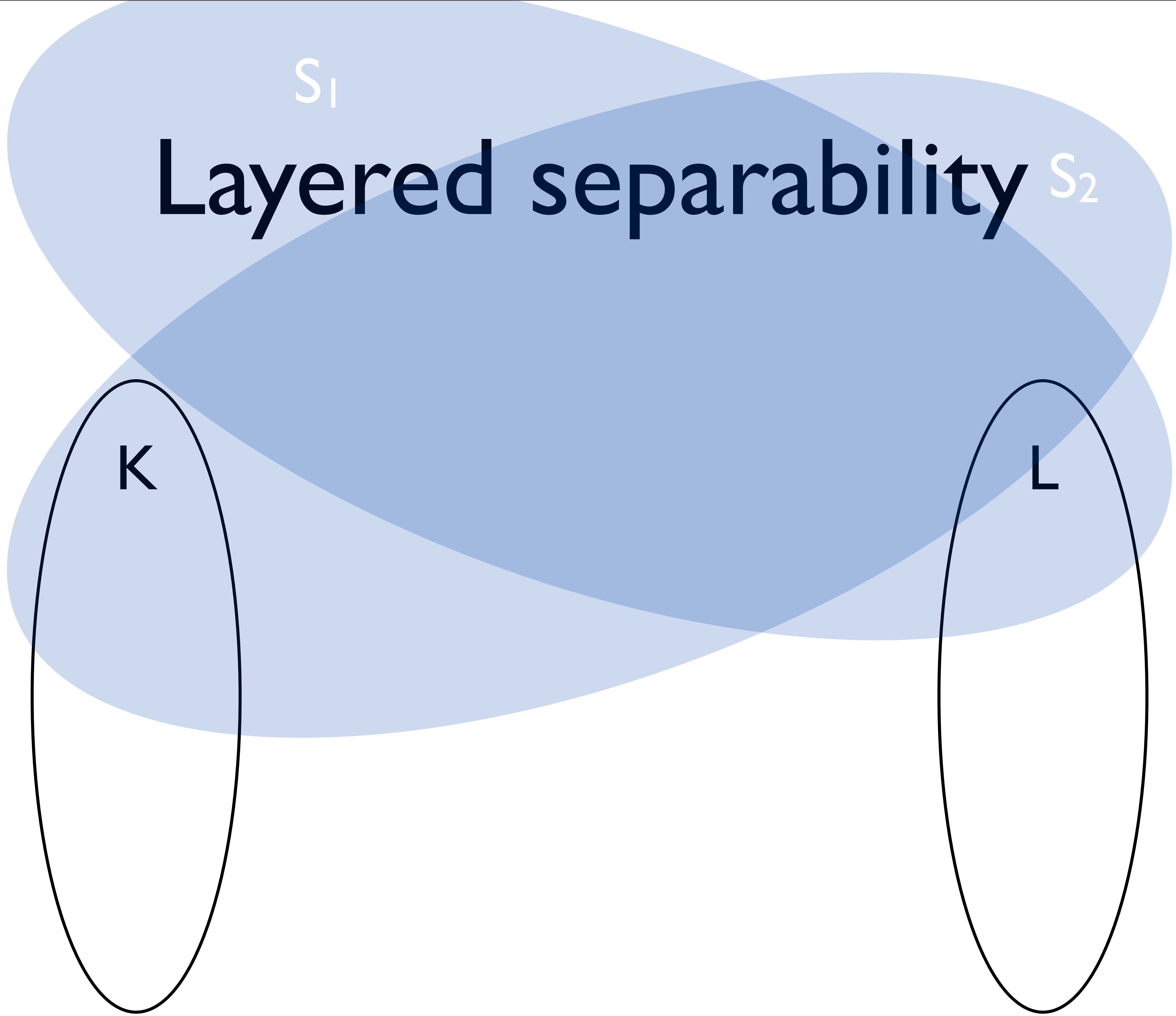
Layered separability

Layered separability

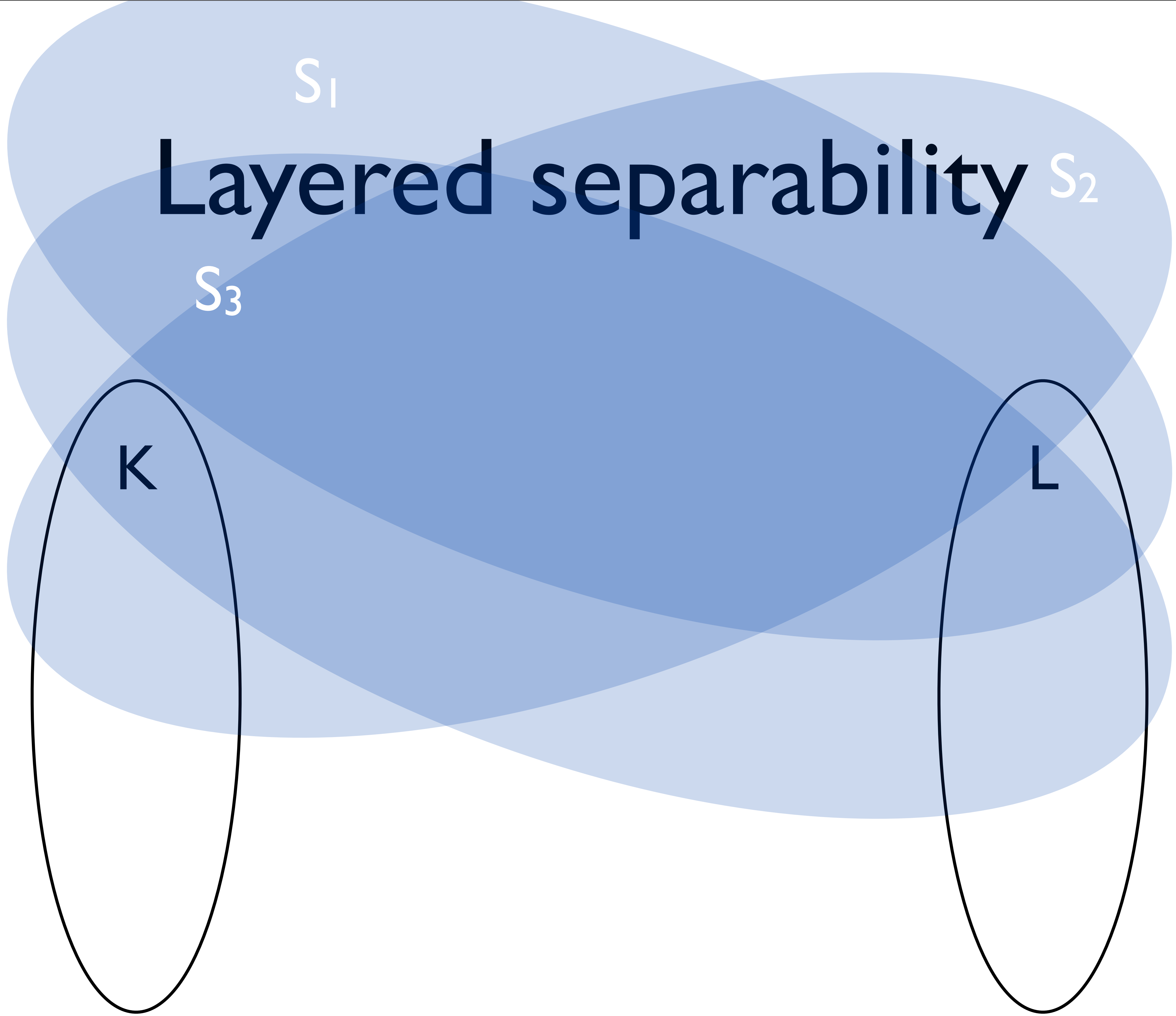
Layered separability



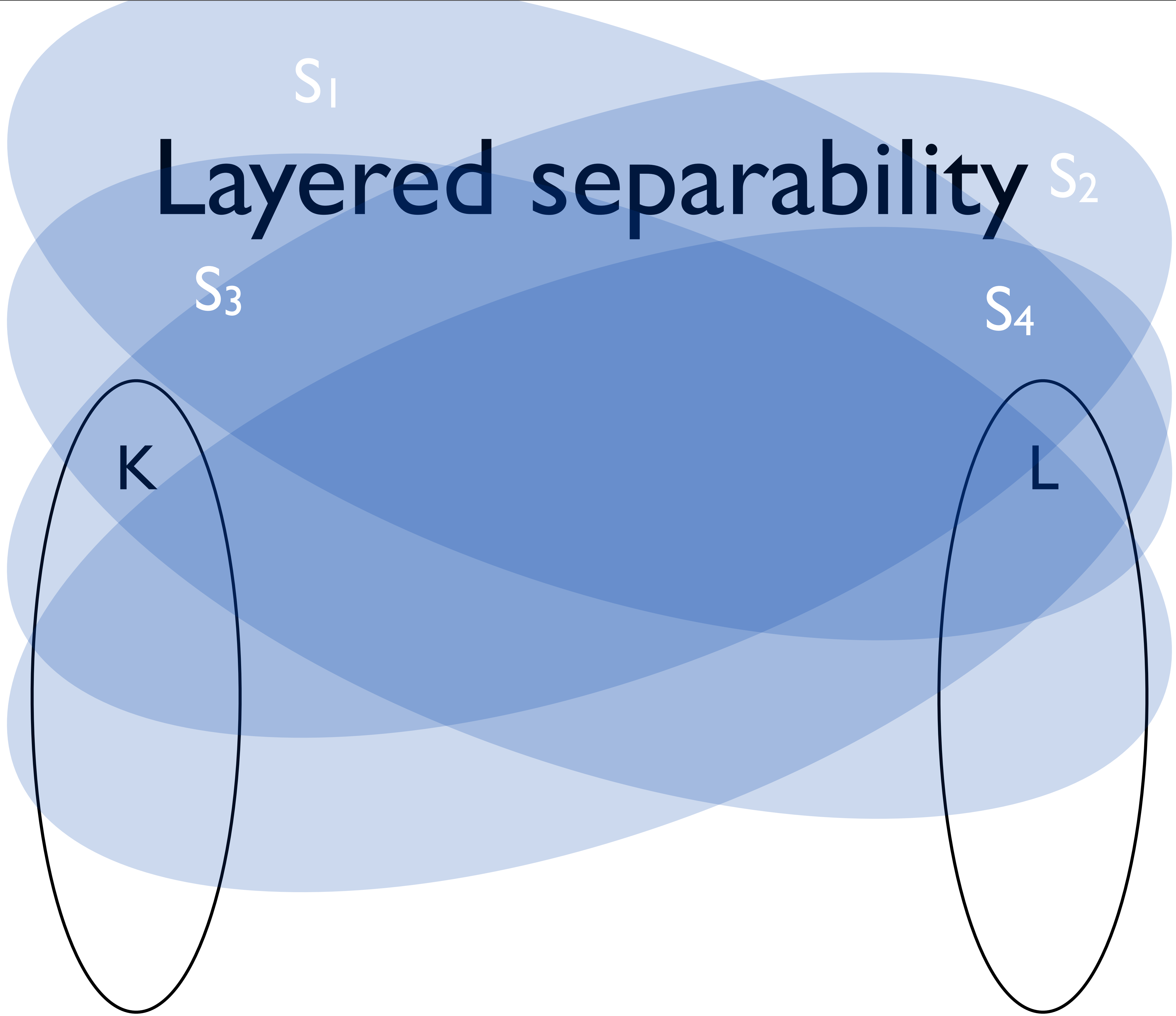
Layered separability



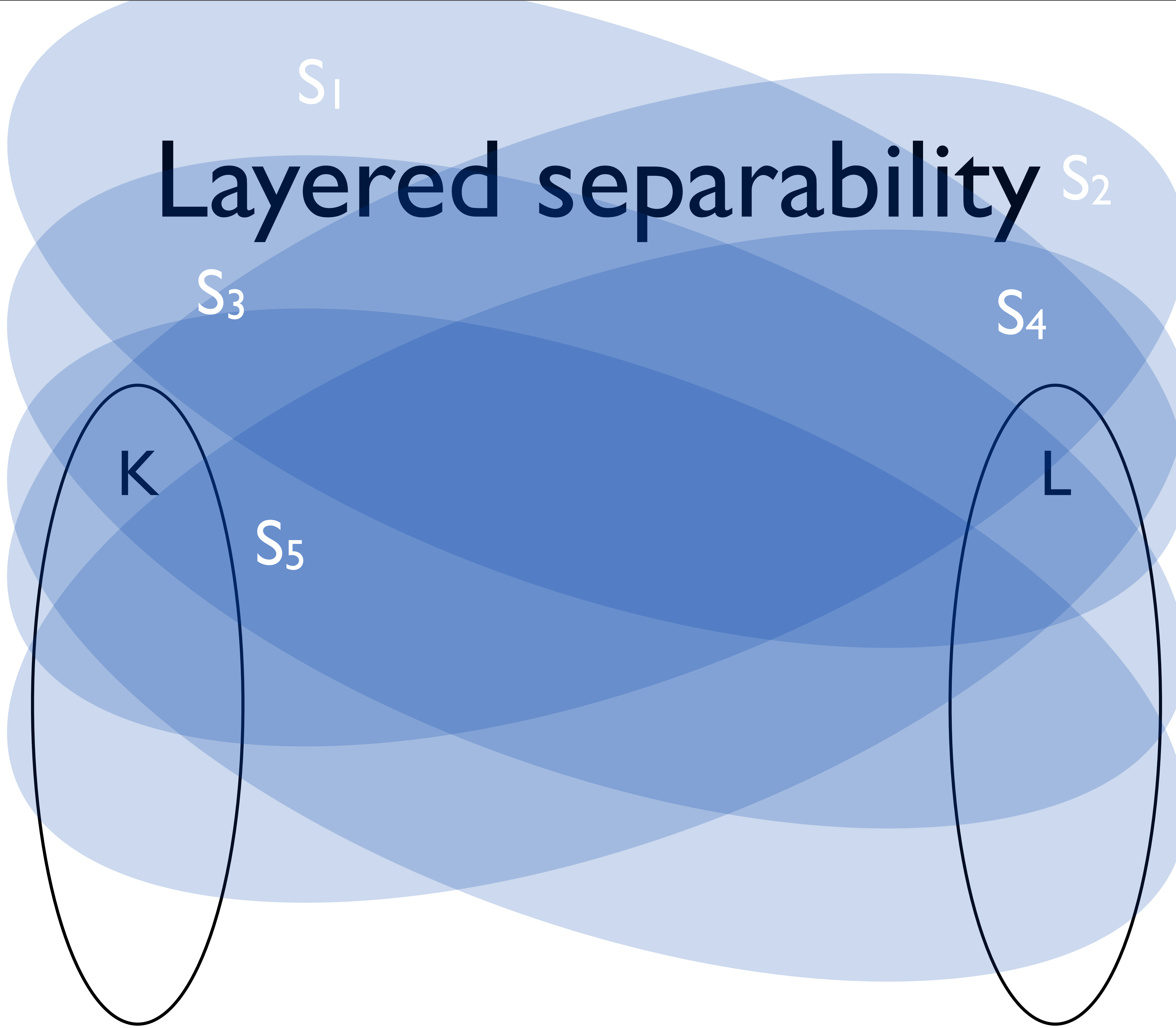
Layered separability



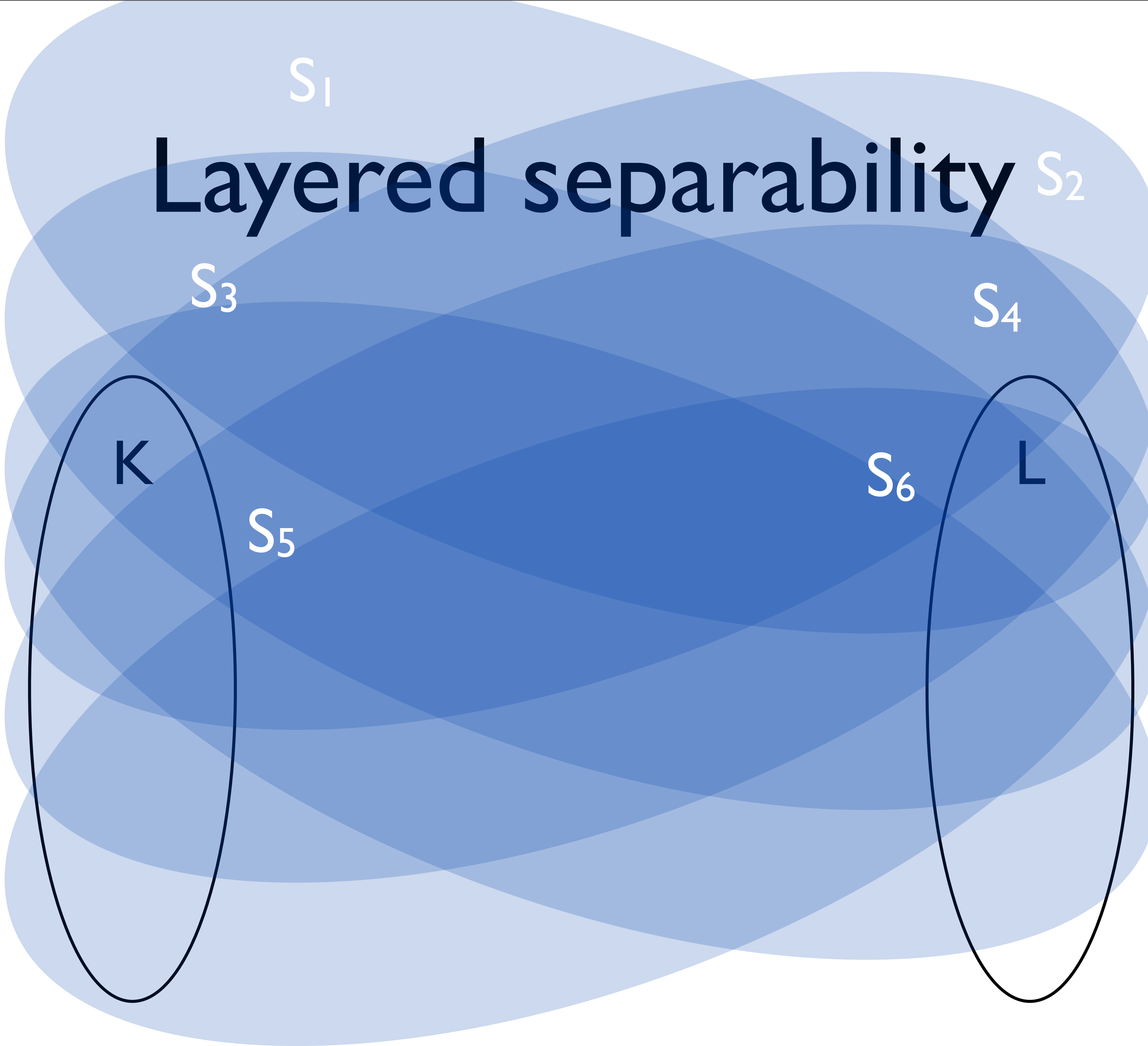
Layered separability



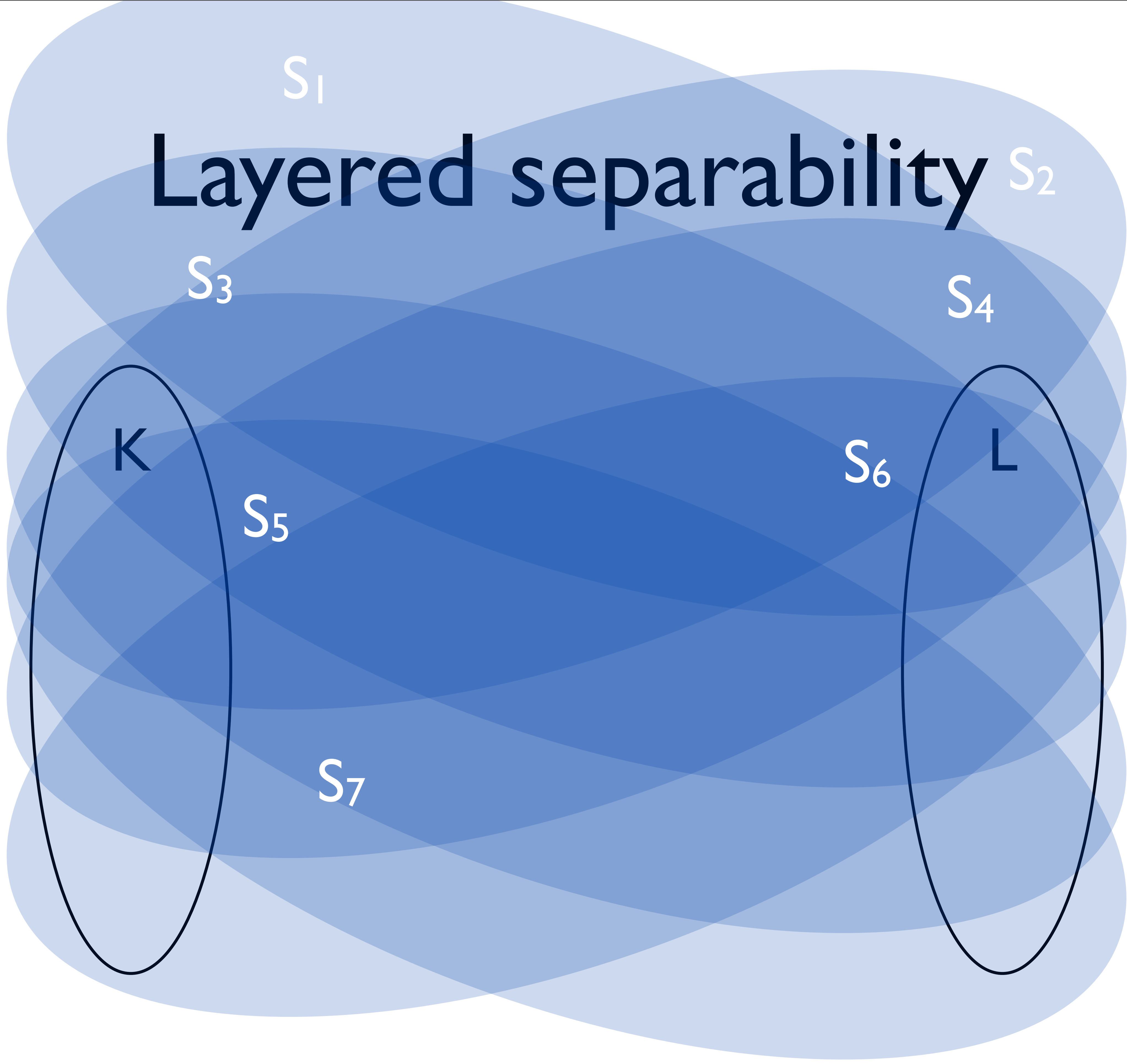
Layered separability



Layered separability



Layered separability



Second main result

For any two word languages K and L the following conditions are equivalent:

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

Well quasi order (WQO)

Well quasi order (WQO)

quasi order $\precsim$ on X is a WQO

Well quasi order (WQO)

quasi order $\precsim$ on X is a WQO

if

Well quasi order (WQO)

quasi order $\precsim$ on X is a WQO

if

for any infinite sequence

$x_1, x_2, \ldots$

elements of X

Well quasi order (WQO)

quasi order $\precsim$ on X is a WQO

if

for any infinite sequence

$x_1, x_2, \ldots$

elements of X

there exists $x_i \precsim x_j$ for some $i < j$

Well quasi order (WQO)

quasi order $\lesssim$ on X is a WQO

if

for any infinite sequence

$x_1, x_2, \ldots$

elements of X

there exists $x_i \lesssim x_j$ for some $i < j$

subsequence order is a WQO (Higman's Lemma)

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

3) K and L are layered separable by pieces

Second main result

For any two ~~word~~ languages K and L the following conditions are equivalent:

1) K and L are separable by bool. comb. of $\lesssim$-closed languages

2) there is no infinite $\lesssim$-zigzag between K and L

3) K and L are layered separable by $\lesssim$-closed lang.

Second main result

For any two ~~word~~ languages K and L the following conditions are equivalent:

1) K and L are separable by bool. comb. of $\precsim$-closed languages

2) there is no infinite $\precsim$-zigzag between K and L

3) K and L are layered separable by $\precsim$-closed lang.

for any well quasi order $\precsim$

Effective characterization

Effective characterization

For two regular languages K and L
the following are equivalent:

Effective characterization

For two regular languages K and L
the following are equivalent:

1) there is an infinite zigzag between K and L

Effective characterization

For two regular languages K and L
the following are equivalent:

1) there is an infinite zigzag between K and L

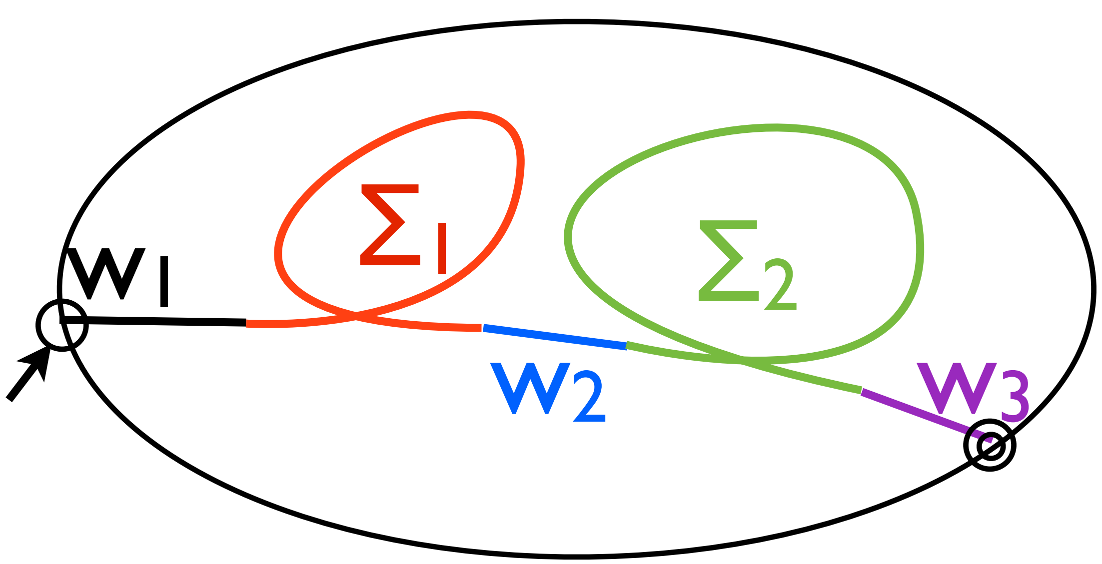
2) in both automata of K and L the following pattern occurs:

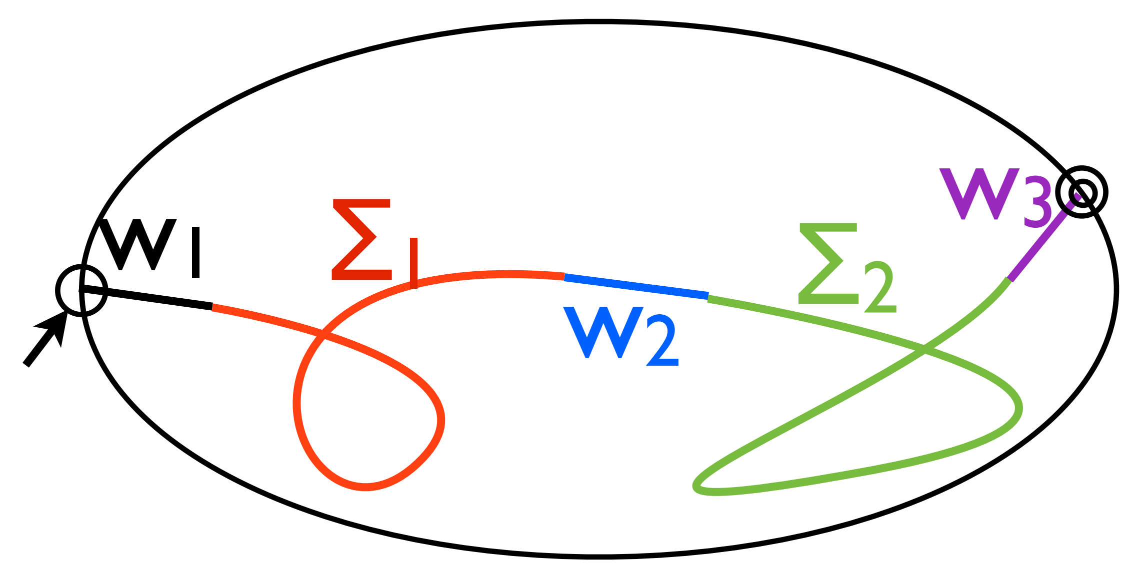
Effective characterization

For two regular languages K and L
the following are equivalent:

1) there is an infinite zigzag between K and L

2) in both automata of K and L the following pattern occurs:



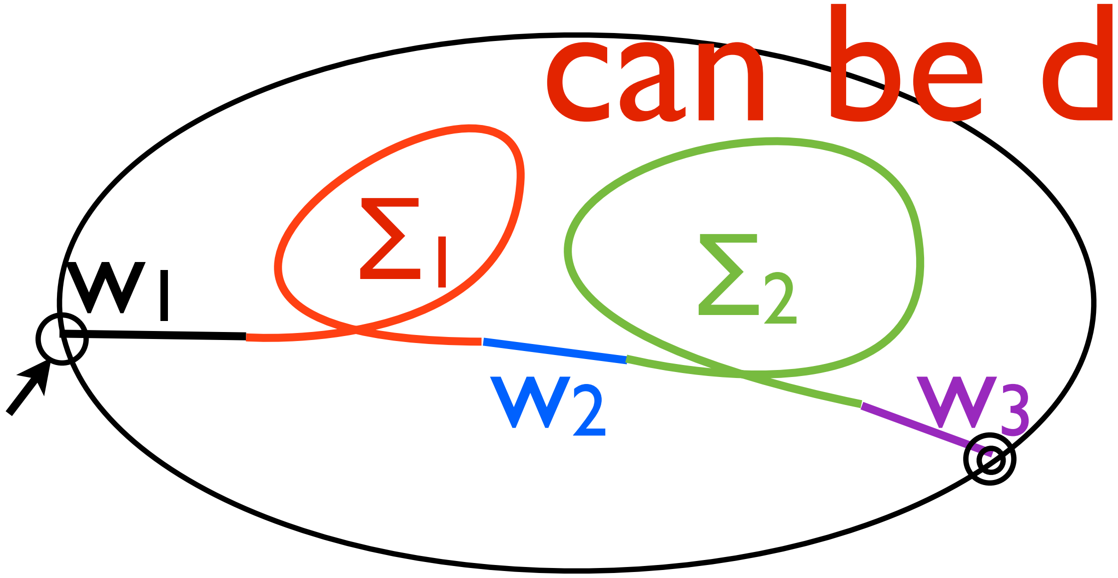


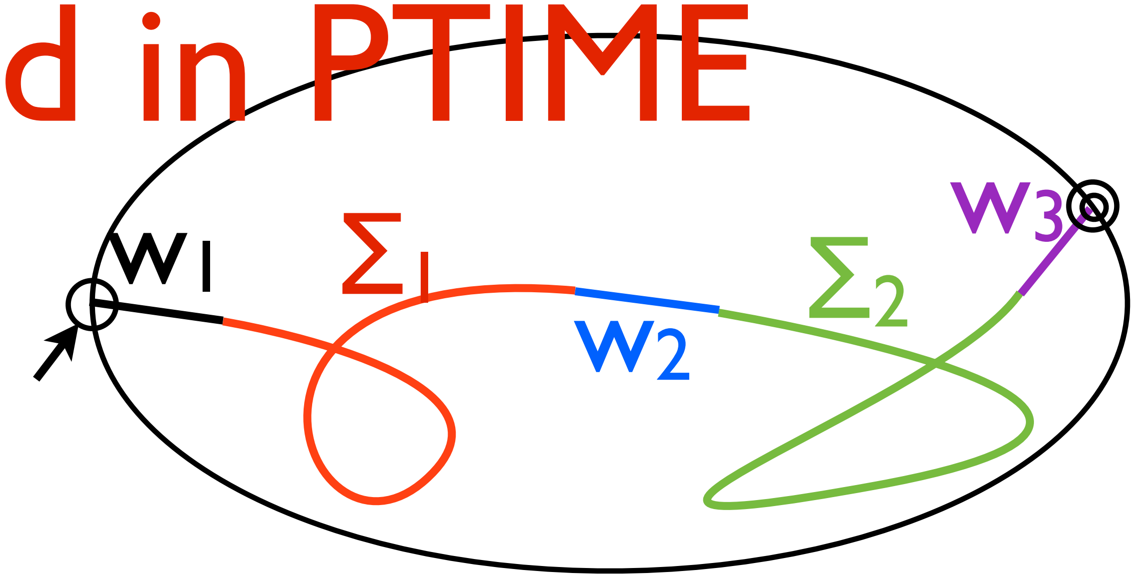
Effective characterization

For two regular languages K and L
the following are equivalent:

1) there is an infinite zigzag between K and L

2) in both automata of K and L the following pattern occurs:





can be decided in PTIME

Another effective characterization

Another effective characterization

Two player game between Zigzag and Doubter

Another effective characterization

Two player game between Zigzag and Doubter

Zigzag tries to create zigzag, Doubter doubts

Another effective characterization

Two player game between Zigzag and Doubter

Zigzag tries to create zigzag, Doubter doubts

Deciding a winner is in PTIME

Another effective characterization

Two player game between Zigzag and Doubter

Zigzag tries to create zigzag, Doubter doubts

Deciding a winner is in PTIME

This approach also solves the tree case

Thank you!