Fixed parameter tractability of non-preemptive multicoloring

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joint work with Dániel Marx

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Non-preemptive sum multicoloring

Input

- graph $G = (V, E)$
- length function $l : V \to \mathbb{N}_+$

Output: scheduling $s : V \to \mathbb{N}$ such that

$$\forall_{\{u,v\} \in E} \quad [s(u); s(u) + l(u)) \cap [s(v); s(v) + l(v)) = \emptyset$$

Objective: minimize sum of completion times

$$\sum_{u \in V} (s(u) + l(u))$$

FPT (treewidth, maximum length)

Halldórsson, Kortsarz (2001)

Non-preemptive sum multicoloring can be solved in

$$O(f(k, p) \cdot n^2)$$

on graphs of treewidth $\leqslant k$ and maximum vertex length $\leqslant p$.

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Proof:

- first $4pk \log n$ colors sufficient
- bottom-up DP on tree decomposition
  - $(4pk \log n)^k$ states in each tree node
- $O((4pk \log n)^k n)$, which is $O((8pk)^k n^2)$

FPT (treewidth, number of different lengths)

$\{l(u) : u \in V\} = \{p_1 < p_2 < \ldots < p_d\}$ i.e. there are d different lengths.

Our result

Non-preemptive sum multicoloring can be solved in

$$O(f(k, d) \cdot n^2)$$

Simple case: two different lengths

Assume $d = 2, p_1 = p, p_2 = 1$.

We will prove that

$$s(u) + l(u) = \alpha p + \beta, \quad \alpha \leqslant 4k \log n, \beta \leqslant (4k \log n)^2.$$

$\implies$ only $(4k \log n)^3$ colors to be considered

Simple case: two different lengths

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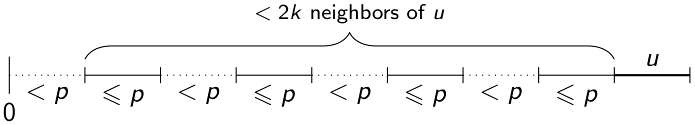
- each of them finishes before $2p \cdot 2k$



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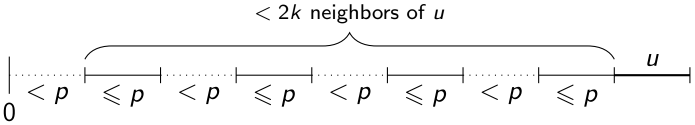

- repeat $\log n$ times

Simple case: two different lengths

Goal:

$$s(u) + l(u) = \alpha p + \beta, \quad \alpha \leqslant 4k \log n, \beta \leqslant (4k \log n)^2.$$

We have already bounded α. If $p \leqslant (4k \log n)^2$, we have also bounded β.

From now on we assume $p > (4k \log n)^2$.

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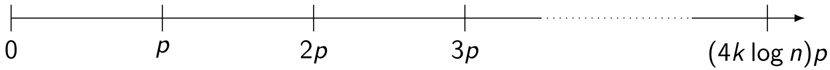

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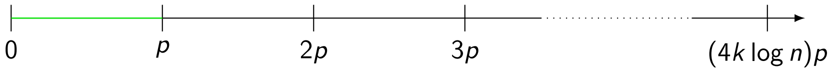

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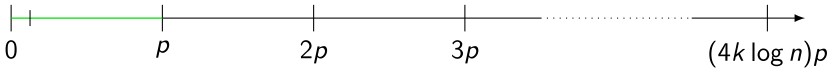

$\vdash\!\dashv = 4k \log n$

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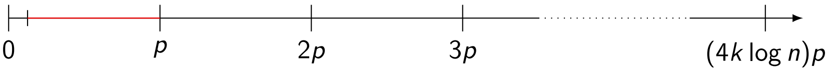

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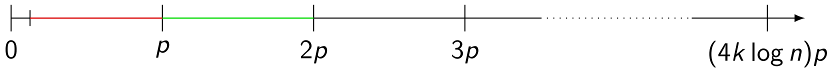

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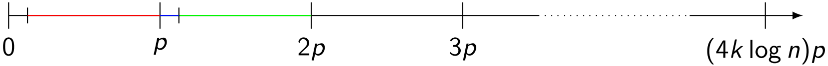
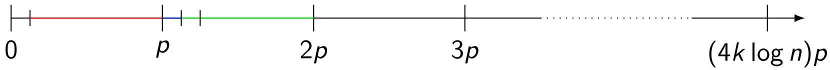


$\vdash\!\dashv = 4k \log n$

— = nothing finishes here

— = big nodes can finish only here

Simple case: two different lengths



$\vdash\!\dashv$ = $4k \log n$

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