

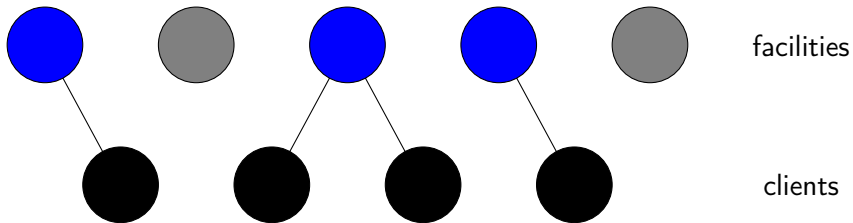
# An Improved Approximation For $k$ -Median Problem

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$k = 3$



**Bi-point solution:** a convex combination of two integral solutions to the LP.

An algorithm:

- Construct a bi-point solution
- Round bi-point solution to integral one

## $k$ -Median Problem

Year	Authors	Bi-point cost	Bi-point rounding	Apx.
'99	Charikar et al.	-	-	$6 + \frac{2}{3}$
'99	Jain & Vazirani	$3 + \epsilon$	2	$6 + \epsilon$
'01	Arya et al.	-	-	$3 + \epsilon$
'02	Jain et al.	$2 + \epsilon$	2	$4 + \epsilon$
'12	Li & Svensson	$2 + \epsilon$	$1.366 + \epsilon$	$2.732 + \epsilon$
'15	Our work	$1.953 + \epsilon$	$1.337 + \epsilon$	$2.611 + \epsilon$

Year	Authors	Lower bound
'98'	Guha & Khuller	$1 + \frac{2}{e}$

## Construction of a bi-point solution

Algorithm	Method	Apx.	Analysis
JMS	Primal Dual	$2 + \epsilon$	factor-revealing LP (difficult)
JMS'	Primal Dual	$1.953 + \epsilon$	factor-revealing LP (simple)

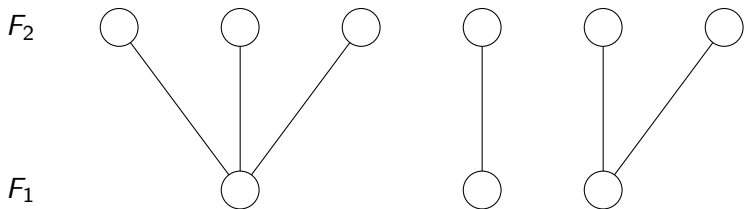
## Bi-point rounding

Again, bi-point solution is a convex combination of two integral solutions  $F_1$  and  $F_2$ , where  $a|F_1| + b|F_2| = k$ .



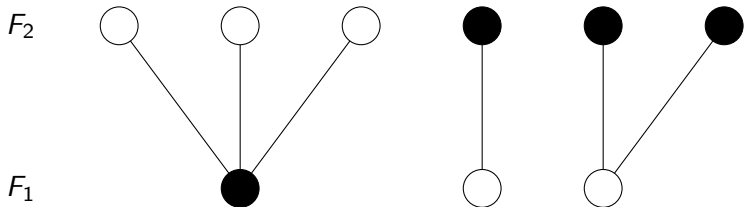
## Bi-point rounding

Form stars by attaching each facility in  $F_2$  to closest facility in  $F_1$ .



## Bi-point rounding

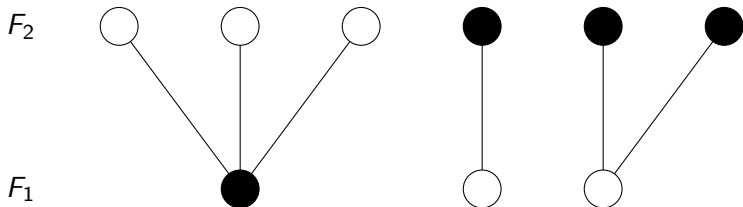
- in  $a$ -fraction of stars open a root
- in  $b$ -fraction of stars open all leafs
- total number of open facilities is  $a|F_1| + b|F_2| = k$





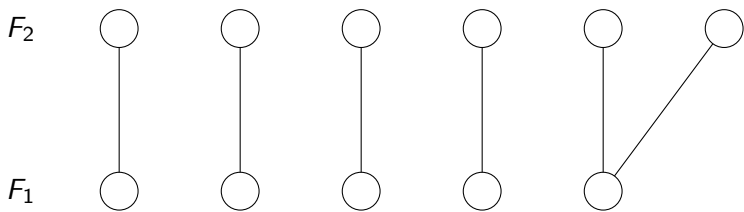
## Bi-point rounding

- every client has "close" open facility
- Li & Svensson get rounding factor 1.366



## Tight instance

- in tight instance almost all stars have exactly one leaf, we exploit this fact to get a factor 1.337



## Open problems:

- A constant factor approximation algorithm for capacitated  $k$ -Median
- Any lower bound better than  $1 + \frac{2}{e}$  for capacitated  $k$ -Median

Thanks! 😊