

# Avoiding Facial Repetitions

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# Result

## Theorem

*Every plane graph is facially non-repetitively  $(O(m^{16}) : m)$ -choosable.*

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## Explanation

- ▶ Given a plane graph,
- ▶ and long  $(O(m^{16}))$  lists for each vertex.
- ▶ We choose shorter  $(m)$  sublists.
- ▶ No facial repetition can be chosen from those sublists.

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Theorem (Przybyło, Schreyer, Škrabuřáková, 2013)

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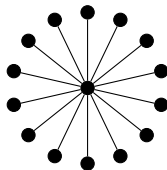
*Every 2-connected plane graph has a bipolar orientation.*

# Bipolar orientations

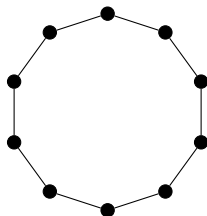
## Definition

- ▶ acyclic
- ▶ single source
- ▶ single sink

Vertex



Face

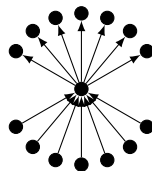


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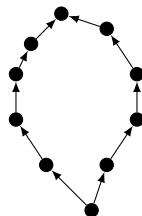
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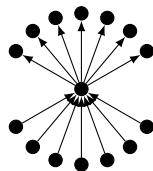


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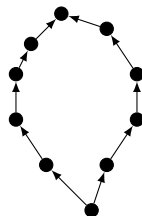
## Applications

- ▶ planarity testing
- ▶ graph drawing

Vertex



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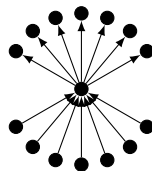


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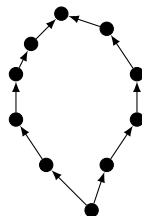
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- ▶ planarity testing
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- ▶ **facially non-repetitive choosability**

## Vertex



## Face





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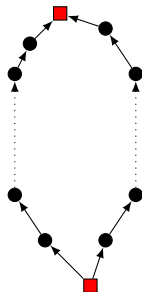
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- ▶ Color left-right graph of faces.

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- ▶ Color left-right graph of faces.
- ▶ Facial filtering.

# Facial filtering

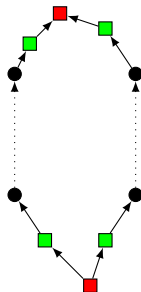
Face



- ▶ Locate *rogue* vertices.

# Facial filtering

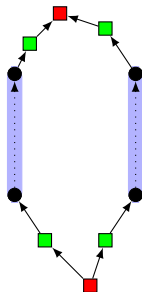
## Face



- ▶ Locate *rogue* vertices.
- ▶ Assign *guardian* vertices.

# Facial filtering

## Face



- ▶ Locate *rogue* vertices.
- ▶ Assign *guardian* vertices.
- ▶ Filter paths.



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- ▶ Facial filtering:  $(5dm^2 : m)$ .
- ▶  $(c^4 5^{15} d^{15} m^{16} : m)$ -choosability.