Datalog and data trees

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Forum Informatyki Teoretycznej 2015

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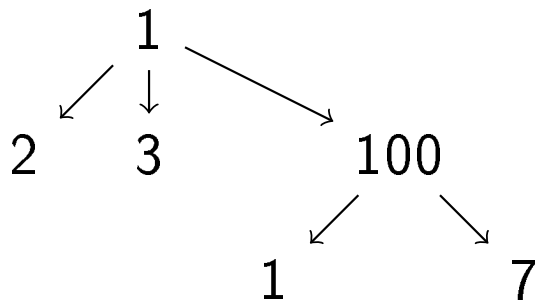
- **intensional** predicates (P, Q);
- one designated **goal** predicate (P).

Trees

Trees over infinte alphabet Σ.
The extensional relations are

- $a(X)$ for each $a \in \Sigma$;
- the label equality $\sim (X, Y)$;
- navigational relations $\downarrow(X, Y)$ and $\downarrow_+(X, Y)$.



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$$P(X) :- \downarrow(X, Y), \downarrow(Y, Z), P(Z), P(Y) \quad \text{monadic, connected}$$
$$Q(X, Y) :- a(Z), \downarrow_+(X, Y) \quad \text{linear}$$

How to evaluate P in a tree?

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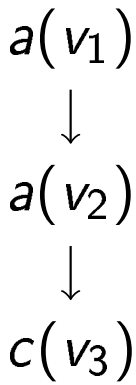

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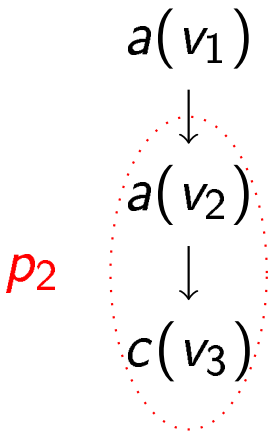

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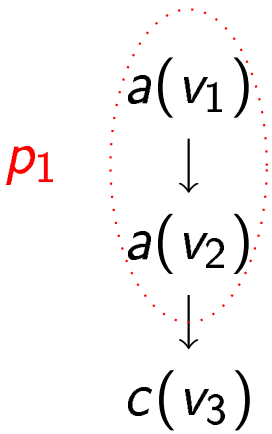

$$P = \{v_1, v_2\}$$

Containment

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Given two datalog programs, $\mathcal{P}$ and $\mathcal{Q}$, is $\mathcal{P} \subseteq \mathcal{Q}$?
That is, does $\forall$ tree t, $P(t) \neq \emptyset \implies Q(t) \neq \emptyset$ hold?

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Containment on trees

- Finite alphabet, monadic programs: decidable (Gottlob, Koch; 2004);
- Infinite alphabet, linear monadic programs: undecidable (Abiteboul, Bourhis, Muscholl; 2013)

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We identified decidable fragments:

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For example:

$$P(X) :- \downarrow(X, Y), a(Y) \qquad \text{downward}$$
$$P(X) :- \downarrow(Y, X), a(Y) \qquad \text{not downward}$$

Summary of the results

	Unranked trees		Ranked trees	
	linear	non-linear	linear	non-linear
D-Datalog($\downarrow, \downarrow_+$)	ExpSpace	2-ExpTime	Undec.	Undec.
Datalog($\downarrow$)	in 3-ExpTime	Undec.	2-ExpTime	in 3-ExpTime

Table : Complexity of containment for datalog fragments.

Conclusion

Thank you for your attention!